**Homework 2**

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**1. Spectral Analysis of Signals**

Consider the signal x(n)=sin(2\*pi\*f1\*n/fs)+sin(2\*pi\*f2\*n/fs), where f1=70, f2=80, fs=2000. Get the frequency spectrum of the signal by using 500-point and 2000-point Discrete Fourier Transform (DFT), respectively. Plot your results graphically and check the difference between the two, and then explain the possible reasons why the spectrum of the two cases is different.

Answer: MATLAB .m files are:



Graphs plot as bellow, the first graph that stands 500-points looks not like 2 spectrums with 70Hz and 80Hz separately but have some influences between each other. The 2nd graph that stands 2000-points looks good. They are different because of spectrum leakage for the first signal sampling, 500-points only sampled 500/2000=0.25s but the sub-signal with f1=70 has cycle of 1/70s which cannot be divided by 0.25s exactly, showing that the original signal was not completely sampled.



Change the f2 to 200 and 200.5, respectively. Calculate the frequency spectrum of the two signals, plot your results and give your solution for improvement.

Answer: MATLAB .m files are:



Graphs plot as bellow, 3 of them have spectrum leakage. We can decrease sampling rate or sample more points to improve.

1. Change fs to 1000, or change N to 1000.
2. Good, nothing should change.
3. Change fs to 250, or change N to 4000.
4. Change fs to 1000, or change N to 4000.



**2. DFT and Convolution**

For a linear system, its input signal and impulse response signal are given by data files ‘xn\_q2.mat’ and ‘hn\_q2.mat’, respectively. Use the standard computation method of convolution to find the response y[n] of the system and represent your results graphically.

Answer: MATLAB code:

xn = load('data/xn\_q2.mat').x;

hn = load('data/hn\_q2.mat').h;

yn1 = conv(xn, hn);

plot(yn1)

legend('yn1')

title('Convolution yn1 of xn and hn')

Graphs plot as bellow:



For the linear system mentioned above, use DFT method to calculate the output signal y[n] and plot all your results graphically, which includes to respectively get the DFT X[f] and H[f] of x[n] and h[n], to compute the Frequency Domain output Y[f] by Y[f]=X[f]×H[f], and then to take the inverse DFT of Y[f] to get the output signal y(n) in time domain.

Answer: MATLAB code:

Xf = fft(xn);

Hf = fft(hn);

Yf = Xf.\*Hf;

yn2 = ifft(Yf);

plot(yn2)

legend('yn2')

title('Using DFT and IDFT')

Graphs plot as bellow:



Plot the output signals of the two different methods in one figure. Check the figure to see if there was any difference between the output signals, and explain the possible reasons. Finally, give your solutions to obtain the same output y(n) using convolution and DFT.

Answer: two curves (yn1 and yn2) are in 1st graph of bellow figure.



We can see that front part of yn2 is different from yn1, and yn2 has only 100 points, but yn1 has 199 point. Because convolution operation of xn(100) and fn(100) will expand 100 point to 199(100+100-1) points while DFT/IDFT method will not, so we need to add zero padding to xn and fn to reach 199 points and then DFT, we can get the same curve(yn3) as showed in the 2nd graph in the above figure.

Code:

figure(2)

subplot(211)

hold on

plot(yn1)

plot(yn2)

legend('yn1','yn2');

hold off

xn2 = [xn zeros(1,99)];

hn2 = [hn zeros(1,99)];

Xf2 = fft(xn2);

Hf2 = fft(hn2);

Yf2 = Xf2.\*Hf2;

yn3 = ifft(Yf2);

subplot(212)

plot(yn3)

legend('yn3');

title('Using DFT and IDFT with 0 Padding')

**3. Frequency Response of a System**

The impulse response h(n) of a system is given by “hn\_q3.mat”, and the sampling frequency fs is 100Hz. Please answer the following questions:

(1) Please plot the amplitude and phase spectra of this system using Matlab. Observe the spectra and answer: What type of system is it? (low-pass, high-pass or band-pass) What is the cut-off frequency? Is it linear phase or non-linear phase within the passband frequency range?



Answer:

1. Band-pass system;
2. The 2 cut-off frequencies: f1≈6.5Hz, f2≈21Hz
3. It’s linear phase.

Code:

hn = load('data/hn\_q3.mat').hn;

Hf=fft(hn);

fs = 100;

figure(2)

subplot(211)

plot(20\*log10(abs(Hf(1:fs/2))))

xlabel('Frequency')

ylabel('Amplitude(dB)')

subplot(212)

plot(unwrap(angle(Hf(1:fs/2))))

xlabel('Frequency')

ylabel('Phase')

(2) For an input signal x(n) saved in “xn\_q3.mat”, please calculate the output y(n) using Matlab. Plot the time waveforms of both x(n) and y(n), and summarize the changes after the signal pass through the system.



Answer: The lower frequency components of xn was removed by the system.

Code:

hn = load('data/hn\_q3.mat').hn;

xn = load('data/xn\_q3.mat').xn;

yn = conv(xn, hn);

hold on;

plot(xn)

plot(yn)

legend('xn','yn');

hold off;

(3) Plot the amplitude spectrum of x(n) and y(n), respectively (please use hamming window prior to FFT). Compare the two spectra and answer: Does it agree with your observations in the time waveform comparison? Does it agree with the frequency response of this system?



Answer: It agrees with my observations as well as the frequency response of this system.

Code:

hn = load('data/hn\_q3.mat').hn;

xn = load('data/xn\_q3.mat').xn;

yn = conv(xn, hn);

fs = 100;

fx = (1:fs/length(xn):fs/2);

fy = (1:fs/length(yn):fs/2);

Xf = fft(xn.\*hann(length(xn))');

Yf = fft(yn.\*hann(length(yn))');

figure(2)

subplot(211)

plot(fx, abs(Xf(1:length(fx))))

title('Xf');

xlabel('Frequency');

ylabel('Amplitude');

subplot(212)

plot(fy, abs(Yf(1:length(fy))))

title('Yf');

xlabel('Frequency');

ylabel('Amplitude');